Isotonic Quantile Regression Averaging for uncertainty quantification of electricity price forecasts

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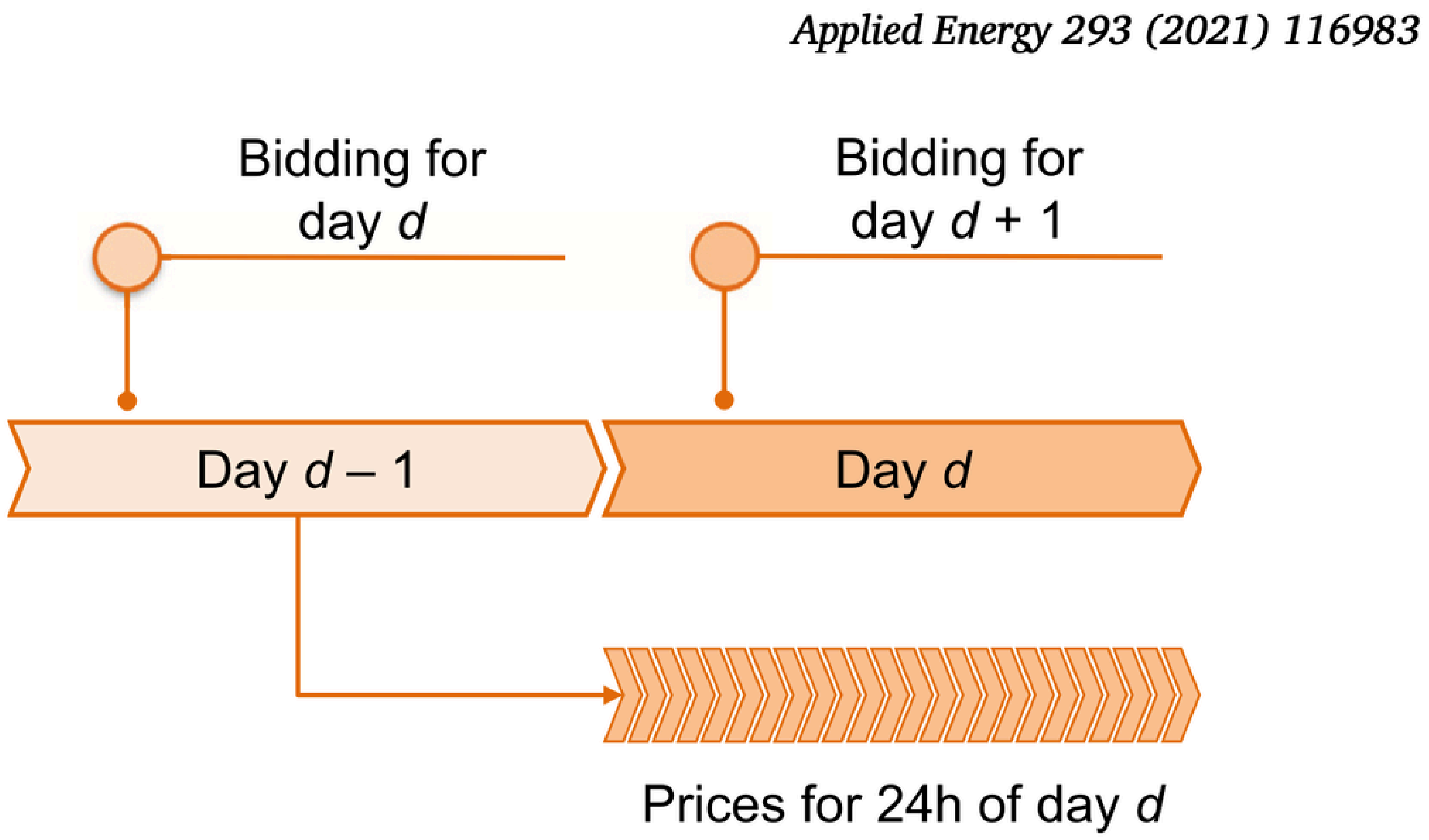
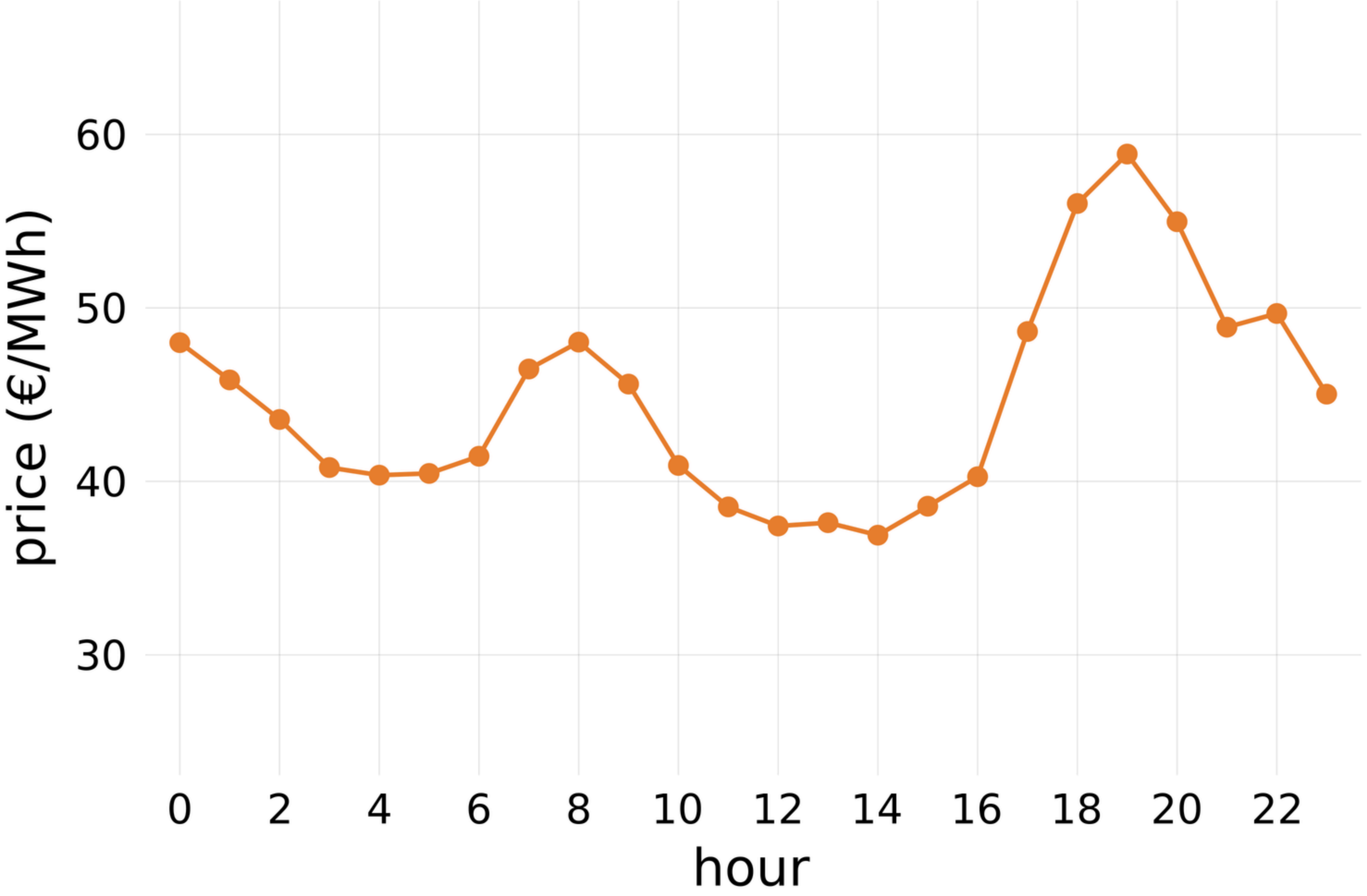
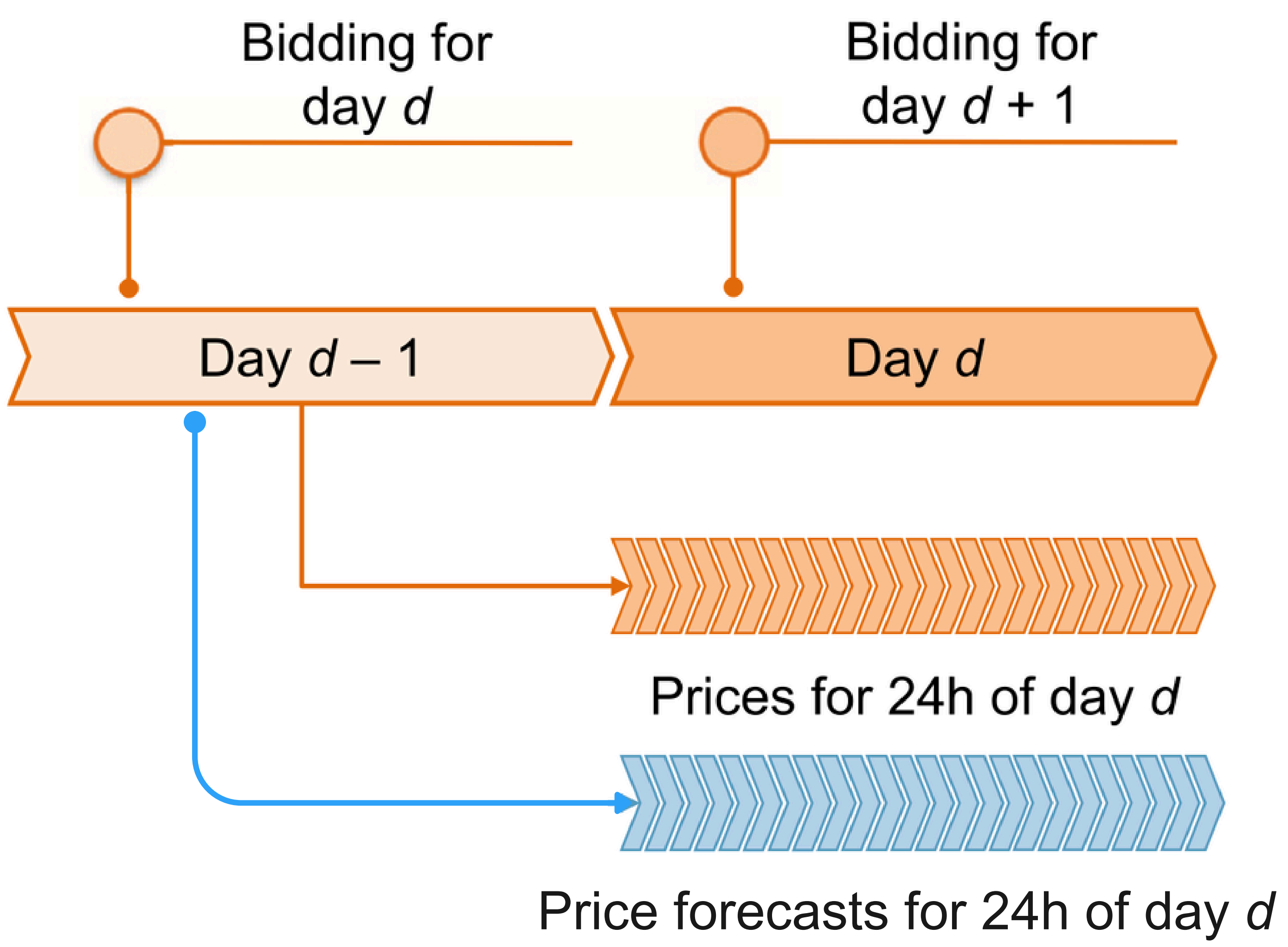
Day-ahead markets

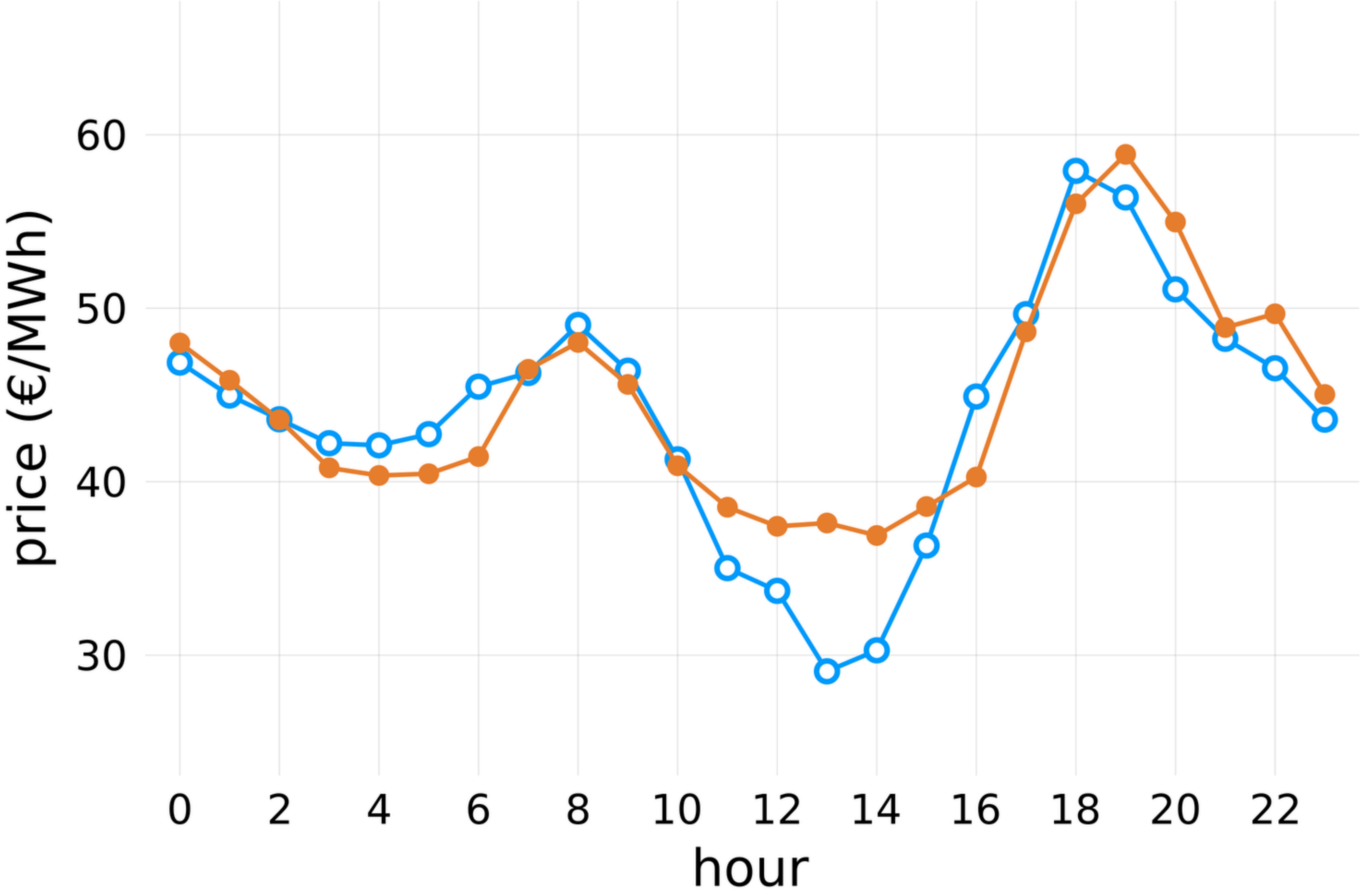




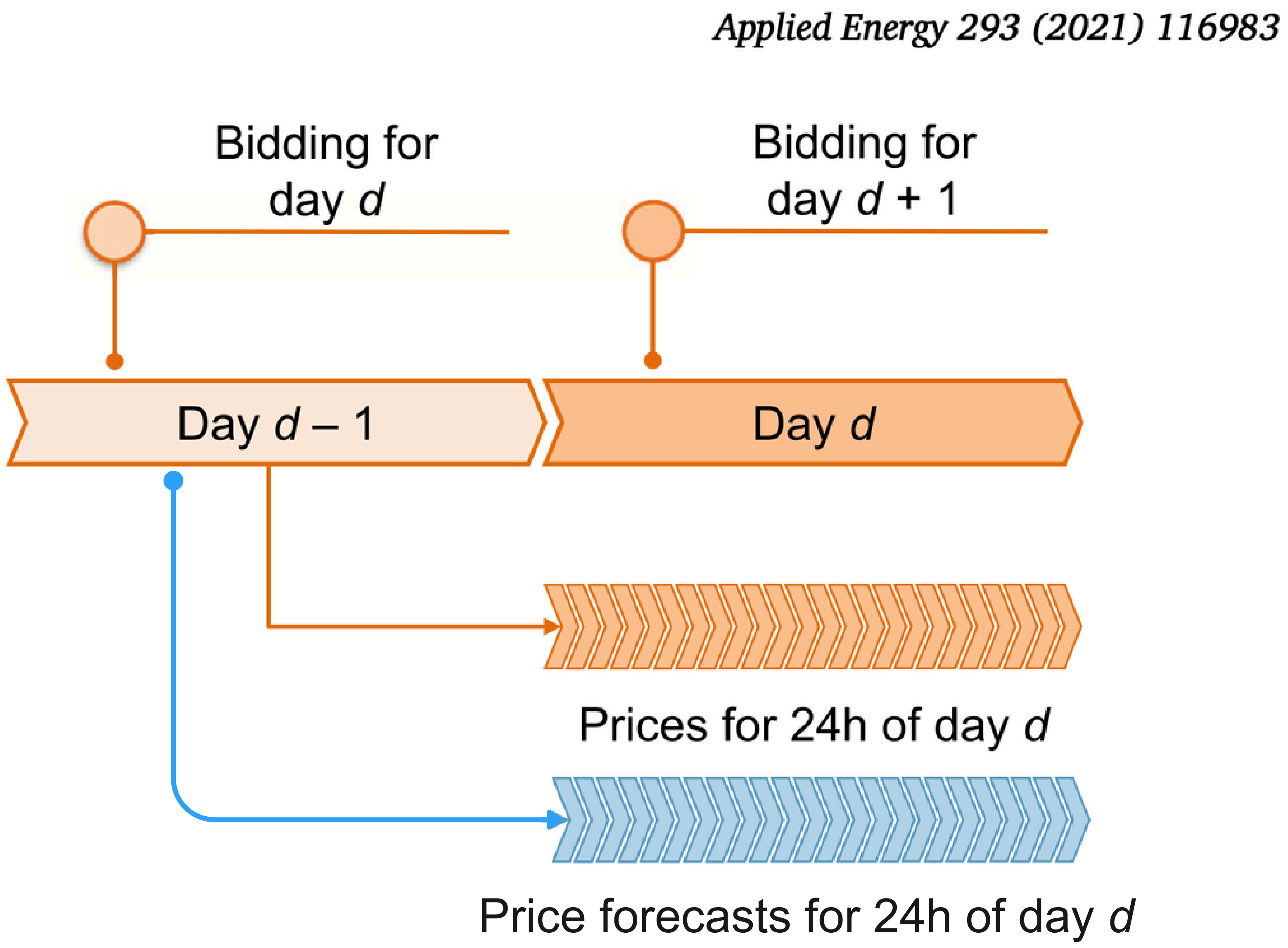
Hourly Forecasts

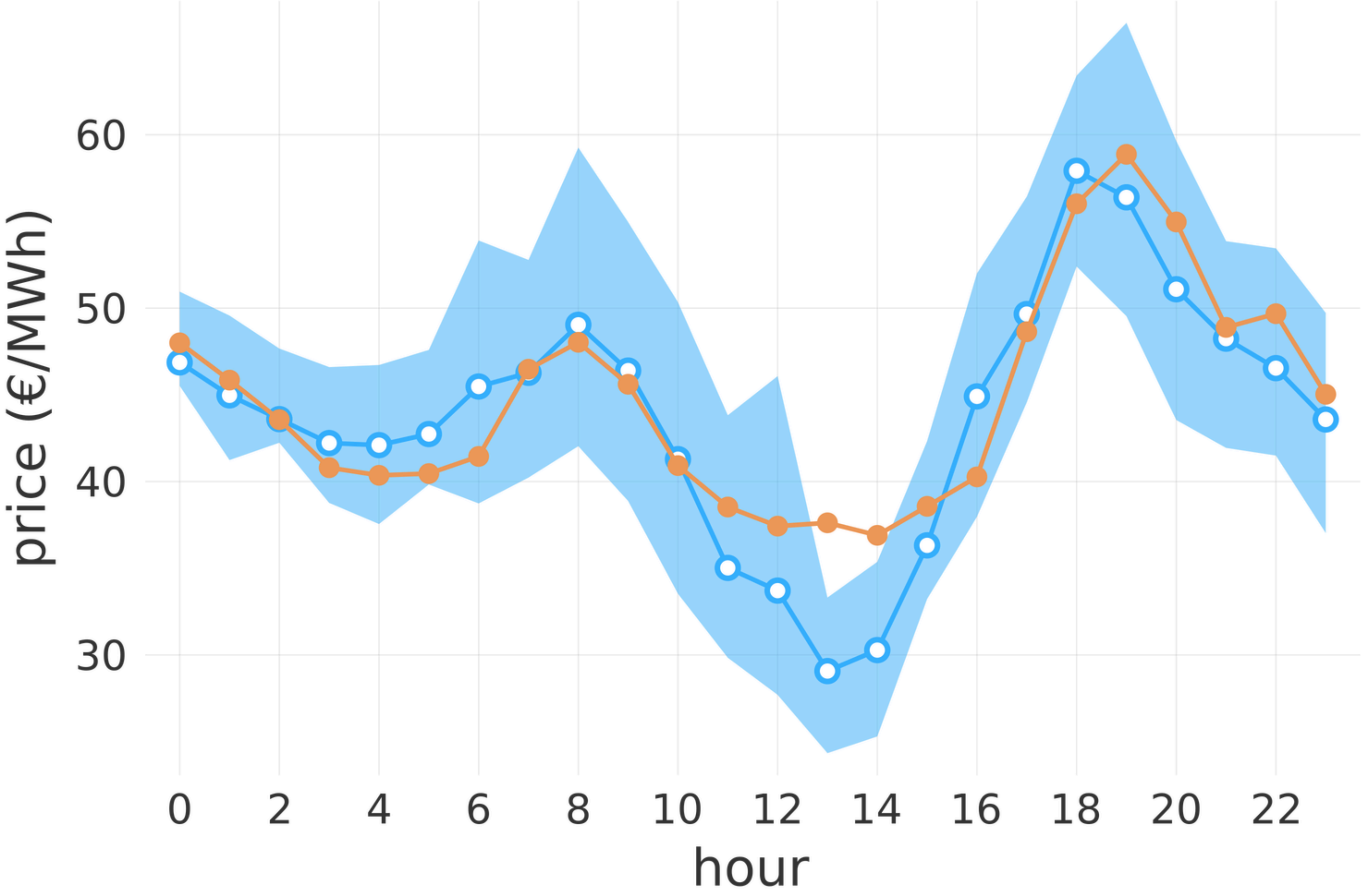
Applied Energy 293 (2021) 116983





Probabilistic Forecasts





Postprocessing of point forecasts

point forecasting model(s)

past prices

exogenous variables

price forecasts

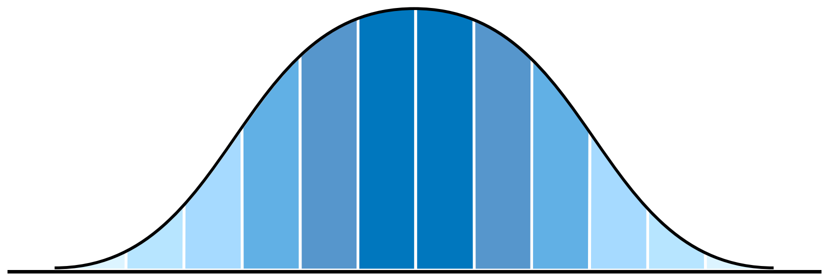
$\hat{p}$

probabilistic forecasting model

$\hat{p}$

$\hat{F}_{\hat{p}}$

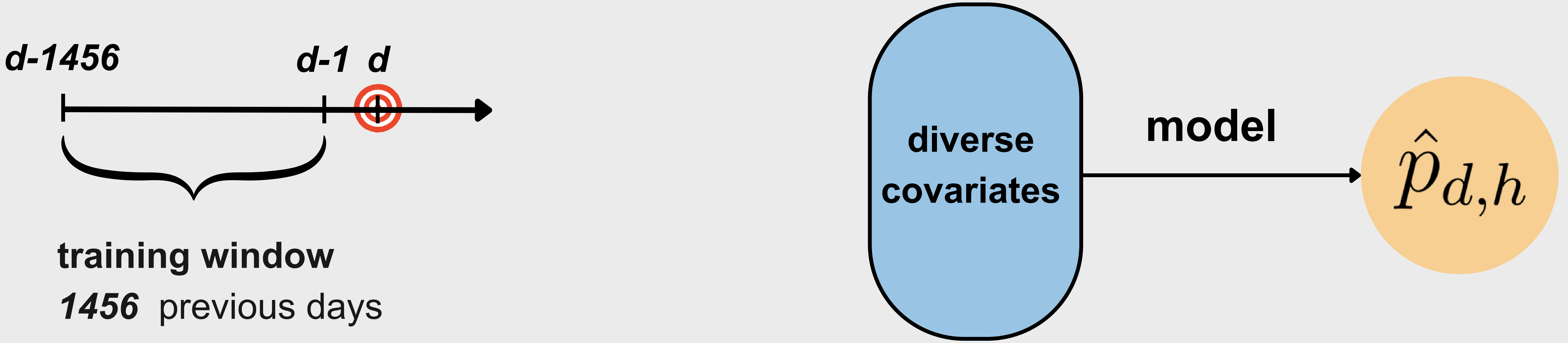
$\hat{Q}_{\hat{p}}(\tau)$



99 percentiles

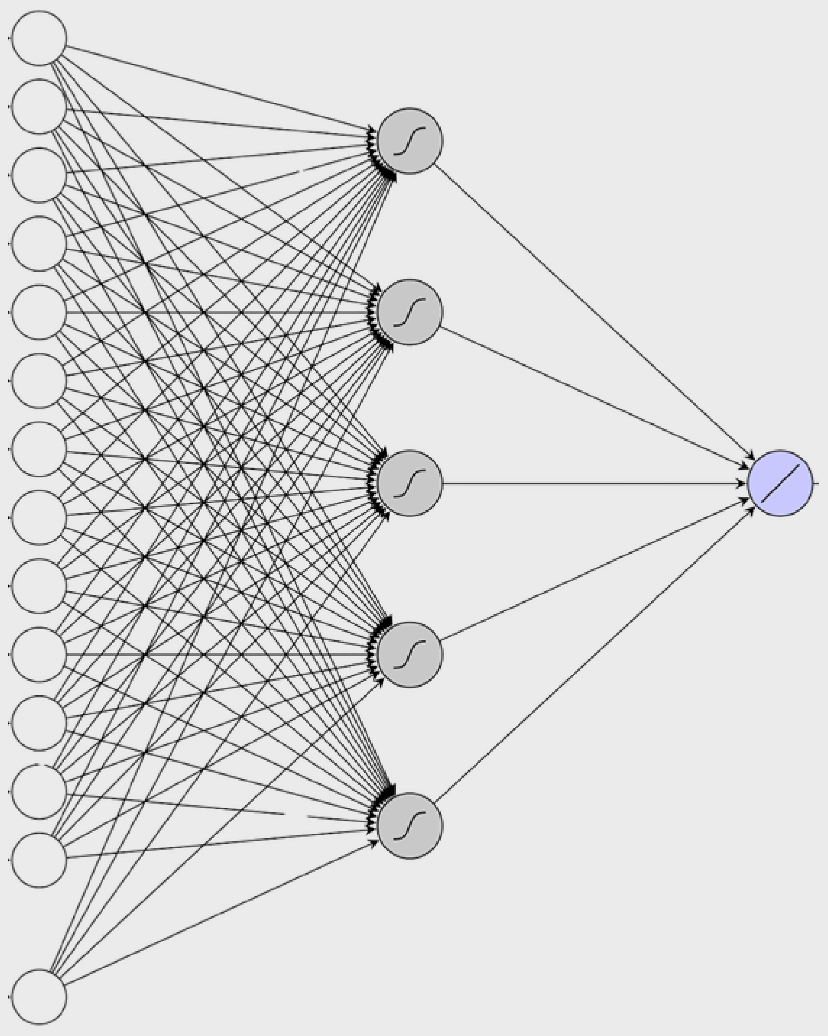
Compute point forecasts

Estimate models for hour ***h*** of day ***d***



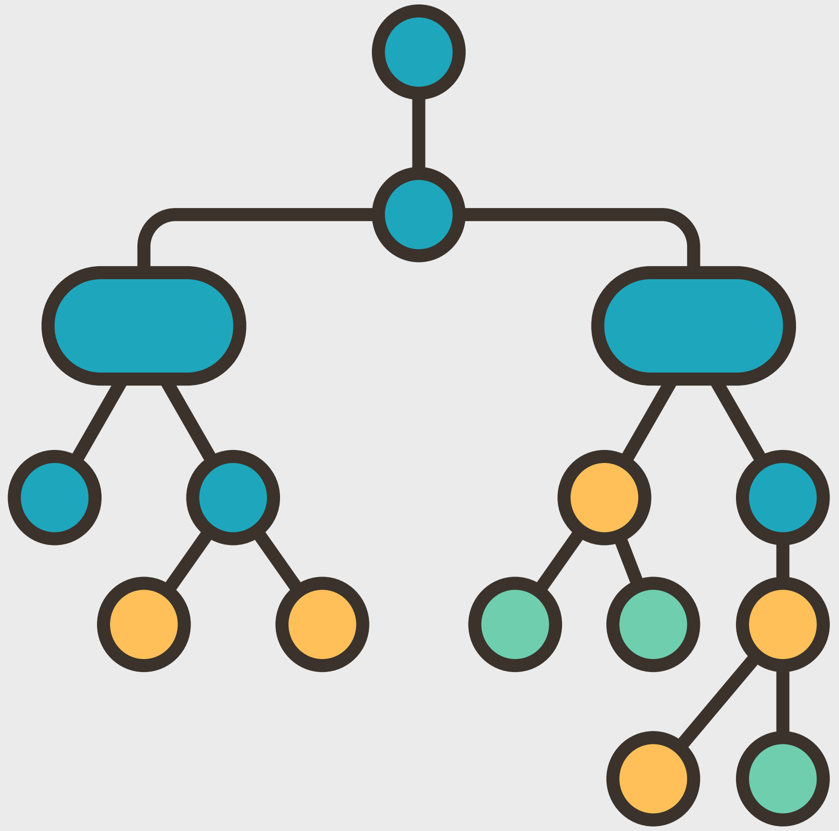
$$\hat{p}_{d,h} = f(\hat{p}_{d-1,h},\ \hat{p}_{d-2,h},\ \hat{p}_{d-7,h},\ \hat{p}_{d-1,24},\ p_{d-1}^{min},\ p_{d-1}^{max},\ Load_{d,h},\ RES_{d,h},$$
$$Coal_{d-2},\ Gas_{d-2},\ Oil_{d-2},\ EUA_{d-2},\ D_d^{(1)}, \ldots,\ D_d^{(7)})$$

Nonlinea**r** **A**uto**R**egressive with e**X**ogenous inputs



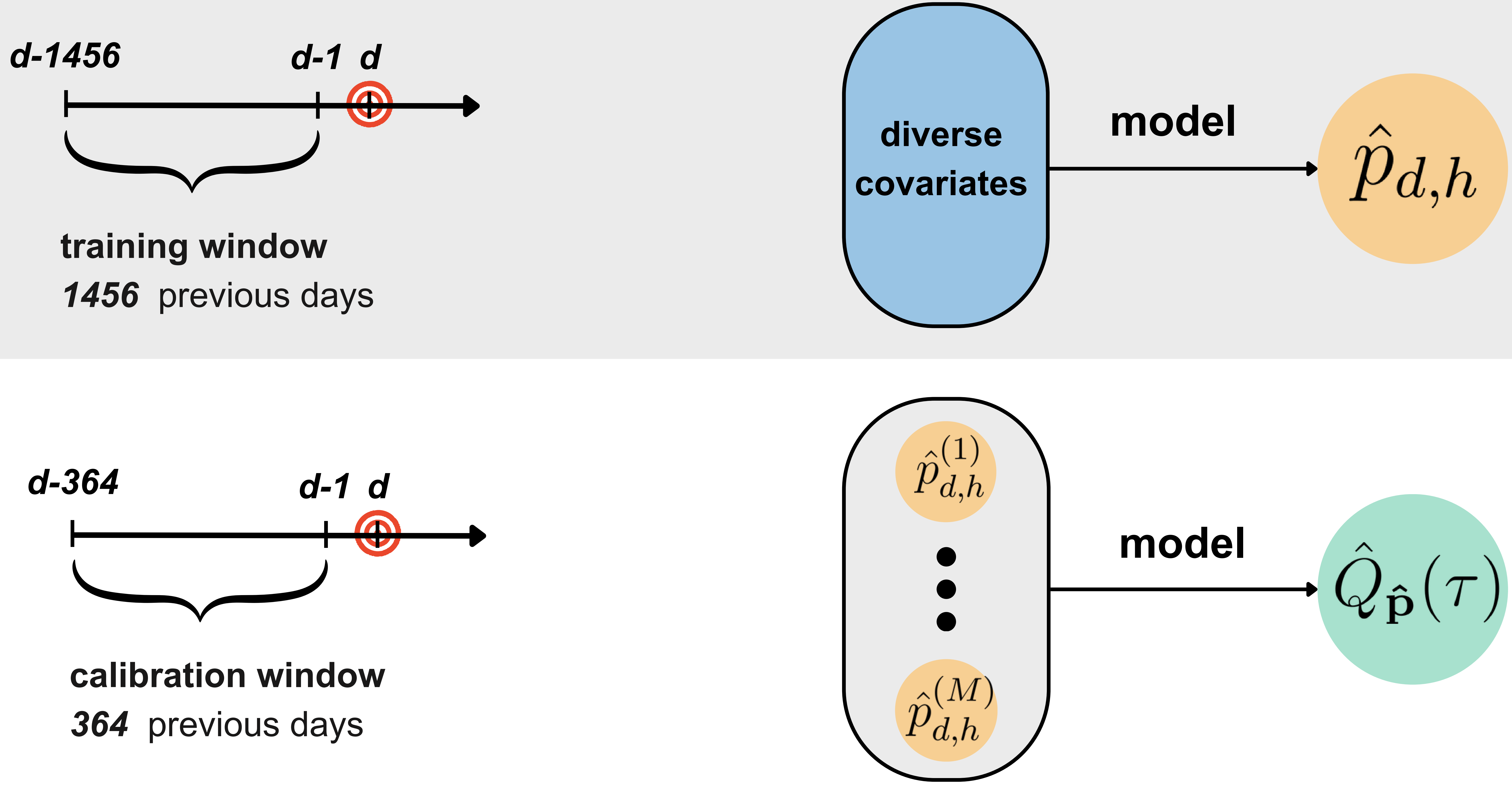
- 5 neurons in the hidden layer
- Hyperbolic tangent activation functions
- Early stopping with 10% validation set
- Estimated by Levenberg-Marquardt
- Ensemble of ***M*** networks
- Random initial parameters and validation set

eXtreme Gradient Boosting



- Python **XGBoost** package
- Mean squared error loss function
- Optuna-based hyperparameter optimization
- ***M*** independent runs for each hour
- Repeated once per year

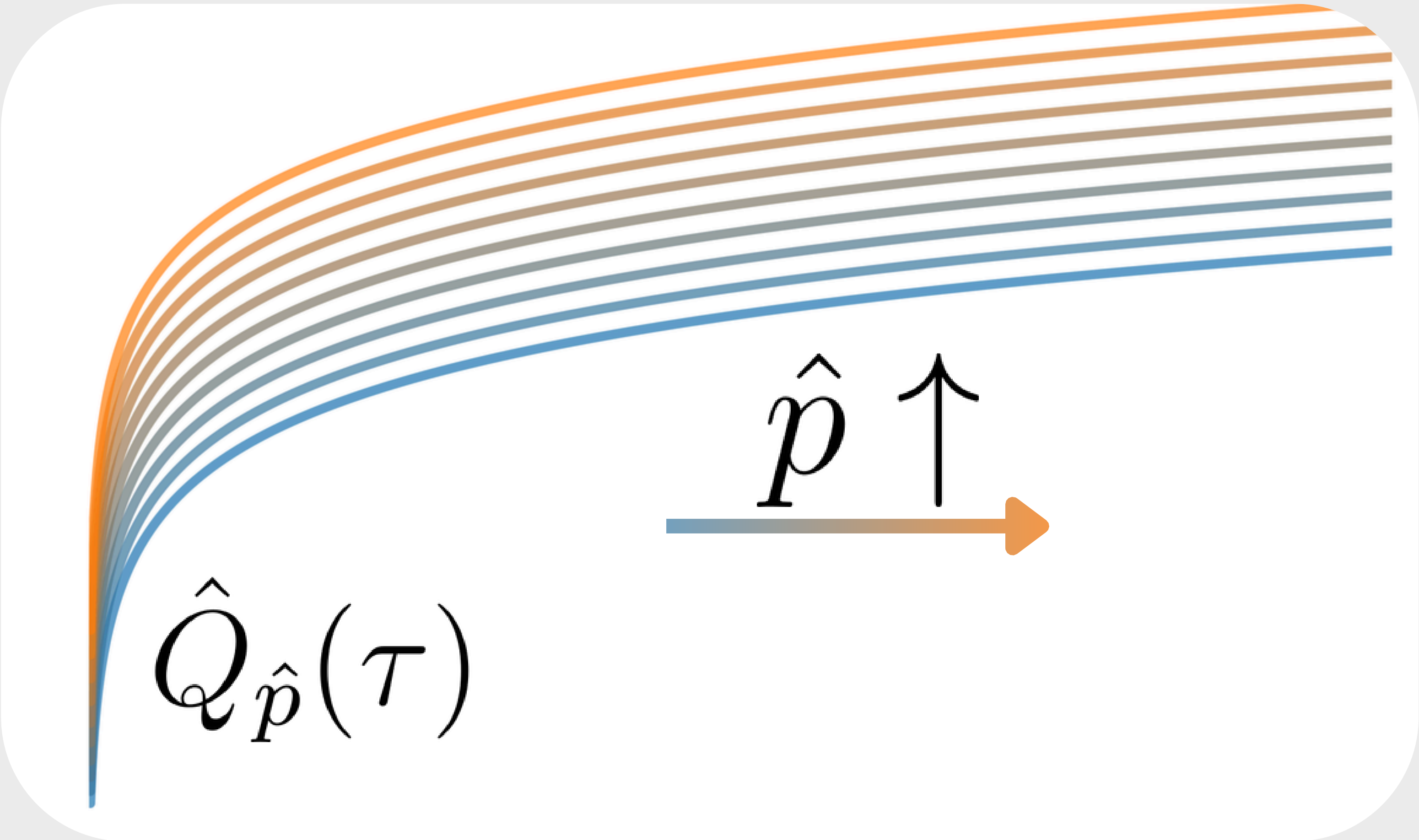
Postprocess point forecasts



Postprocessing methods

- **Conformal Prediction (CP)**
- **Historical Simulation (HS)**
- **Isotonic Distributional Regression (IDR)** (Henzi et al. 2021)

$\hat{Q}_{\hat{p}}(\tau)$ is isotonic (non-decreasing) in $\hat{p}$

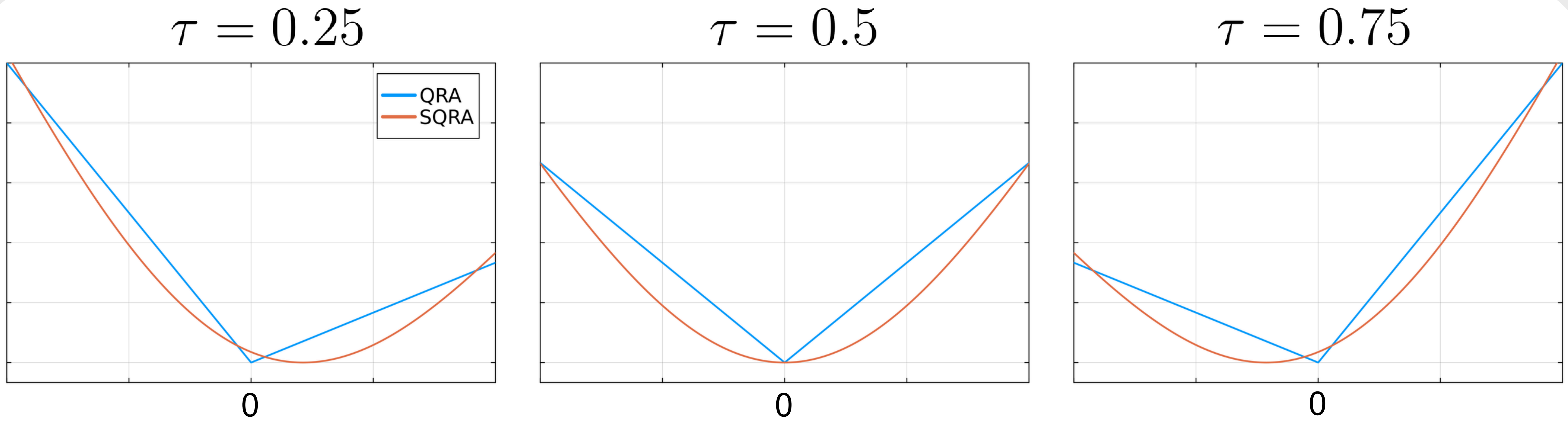


Quantile Regression Averaging (QRA)

$$\hat{Q}_{p_{d,h}}(\tau) = \beta_0 + \beta_1 \hat{p}_{d,h}^{(1)} + \ldots + \beta_M \hat{p}_{d,h}^{(M)}$$

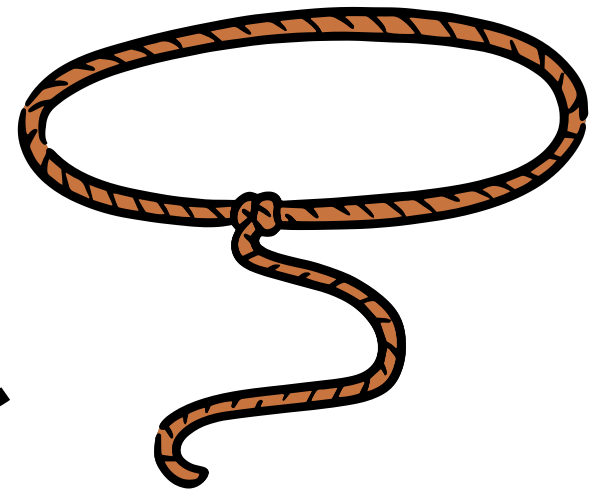
Smoothed Quantile Regression Averaging (SQRA)

Smoothing pinball loss with normal distribution kernel



Lasso Quantile Regression Averaging

$$\hat{Q}_{p_{d,h}}(\tau) = \beta_0 + \beta_1 \hat{p}_{d,h}^{(1)} + \ldots + \beta_M \hat{p}_{d,h}^{(M)}$$

Minimize  + λ 

Regularization path: $\lambda \in [10^{-5}\lambda_{max}, \lambda_{max}]$

20 values distributed equally on a log-scale grid

Best λ **selected by Bayesian Information Criterion**

Isotonic [Smoothed] Quantile Regression Averaging (i[S]QRA)

$$\hat{Q}_{p_{d,h}}(\tau) = \beta_0 + \boxed{\beta_1}\hat{p}_{d,h}^{(1)} + \ldots + \boxed{\beta_M}\hat{p}_{d,h}^{(M)} \qquad \boxed{\beta_i} \geq 0$$

QRA

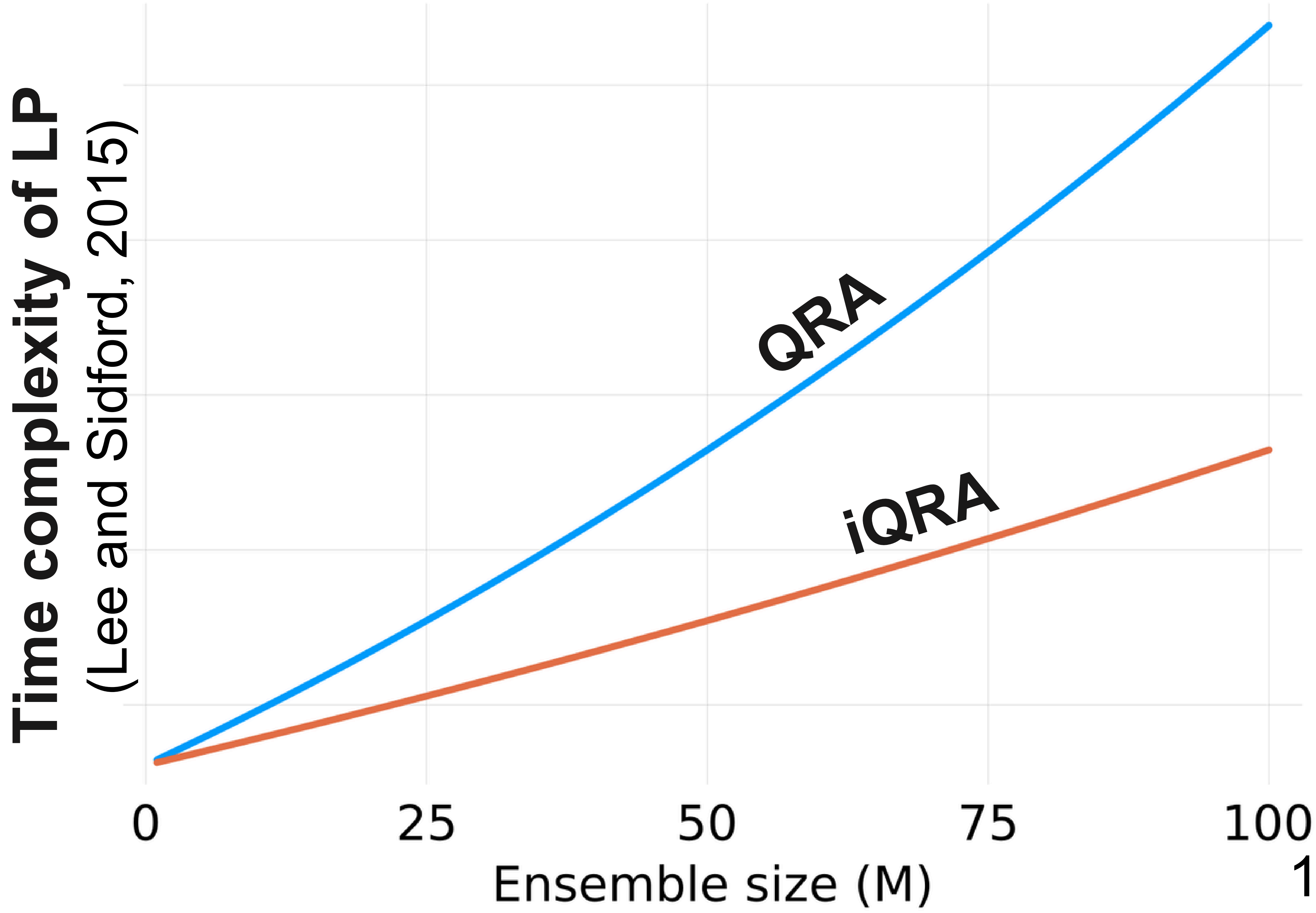
$$\beta_i = \beta_i^+ - \beta_i^-$$

2T + 2 + ***2M*** variables in LP

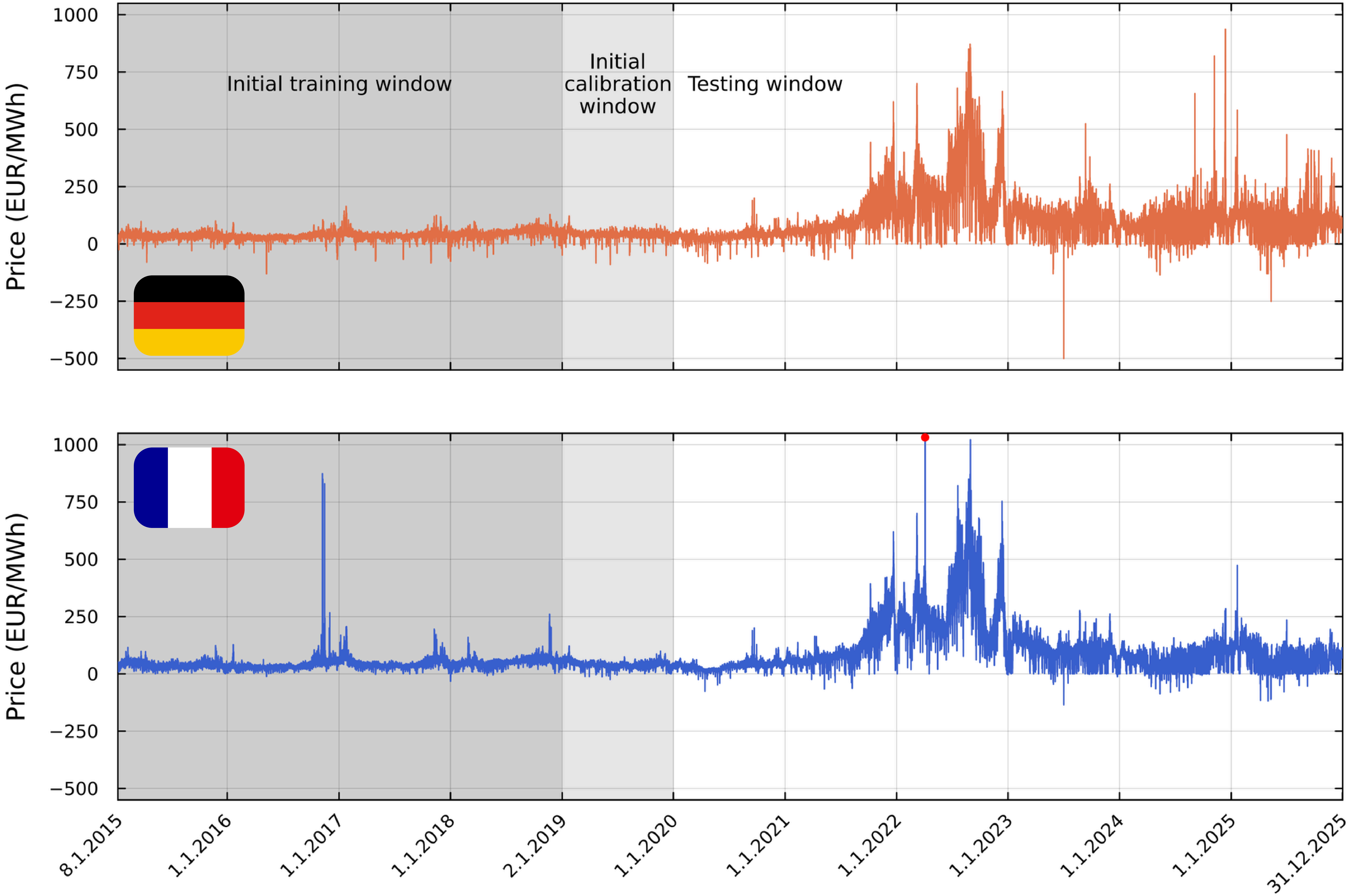
iQRA

$$\beta_i = \beta_i^+$$

2T + 2 + ***M*** variables in LP



Data



Initial training window
Initial calibration window
Testing window
Price (EUR/MWh)
1000
750
500
250
0
−250
−500
8.1.2015
1.1.2016
1.1.2017
1.1.2018
2.1.2019
1.1.2020
1.1.2021
1.1.2022
1.1.2023
1.1.2024
1.1.2025
31.12.2025

Evaluation

Continuous **R**anked **P**robability **S**core (CRPS) - strictly proper scoring function

$$CRPS(\hat{F}, y) = \int (\hat{F}(z) - \mathbf{1}\{y \leq z\})^2 dz$$

CRPS can be approximated by the sum of Pinball loss across the quantiles

$$CRPS(\hat{F}, y) \approx \frac{1}{99} \sum_{i=1}^{99} Pinball(\hat{Q}(\tau = \tfrac{i}{100}), y)$$

CRPS

	Germany			France		
M	**10**	**25**	**50**	**10**	**25**	**50**
HS	5.59	5.54	5.53	5.96	5.90	5.89
CP	5.59	5.54	5.53	5.98	5.93	5.92
IDR	5.43	5.40	5.40	5.88	5.85	5.86
QRA	5.46	5.63	5.97	5.92	6.13	6.58
SQRA	5.44	5.61	5.97	5.90	6.11	6.58
QRM	5.40	5.35	5.34	5.83	5.77	5.76
SQRM	5.40	5.35	5.33	5.82	5.76	5.75
LQRA	5.34	5.26	5.24	5.78	5.73	5.71
iQRA	5.34	5.28	5.25	5.78	5.73	5.71
iSQRA	5.34	5.27	5.25	5.77	5.72	5.70

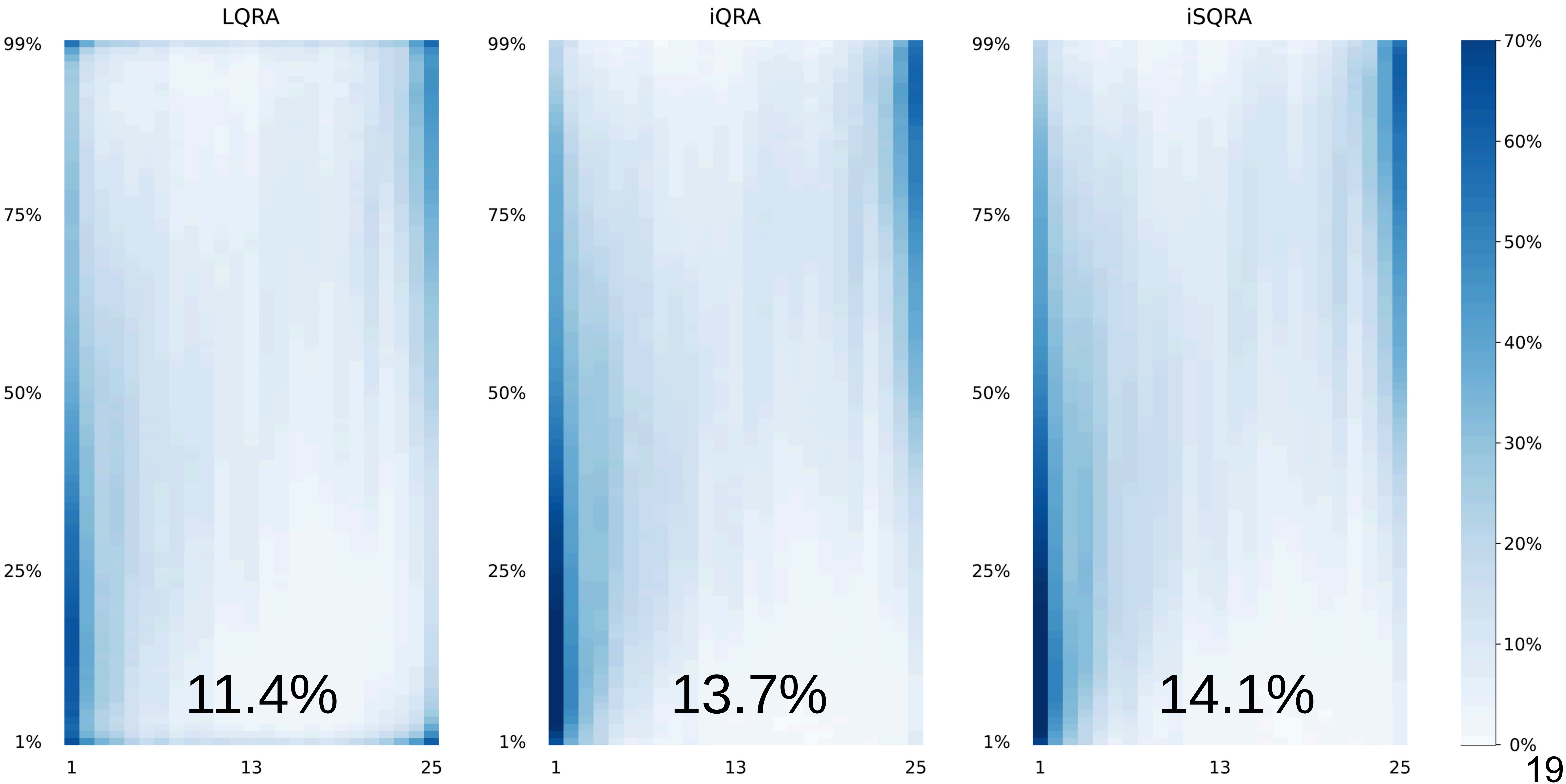
Average pinball for extreme quantiles

	Germany				France			
PI	**1-99%**	**2-98%**	**5-95%**	**10-90%**	**1-99%**	**2-98%**	**5-95%**	**10-90%**
HS	0.93	1.45	2.59	3.93	0.95	1.51	2.73	4.15
CP	0.92	1.44	2.6	3.93	0.94	1.5	2.74	4.18
IDR	0.97	1.4	2.39	3.65	1.09	1.56	2.64	3.96
QRA	0.99	1.42	2.5	3.81	1.07	1.54	2.72	4.15
SQRA	0.99	1.42	2.48	3.79	1.06	1.53	2.69	4.11
QRM	0.77	1.25	2.33	3.64	0.8	1.31	2.47	3.87
SQRM	0.76	1.24	2.33	3.63	0.79	1.3	2.46	3.86
LQRA	0.71	1.16	2.18	3.46	0.78	1.25	2.36	3.75
iQRA	0.71	1.16	2.2	3.49	0.76	1.24	2.37	3.76
iSQRA	0.7	1.15	2.19	3.48	0.75	1.23	2.36	3.75

Average coverage error for prediction interval

	Germany				France			
PI	**1-99%**	**2-98%**	**5-95%**	**10-90%**	**1-99%**	**2-98%**	**5-95%**	**10-90%**
HS	2.0	2.7	4.2	5.1	2.1	2.8	3.9	4.8
CP	1.4	2.2	3.6	4.6	1.6	2.3	3.5	4.2
IDR	7.1	7.5	7.6	7.3	6.9	7.1	7.1	6.6
QRA	7.1	6.8	7.4	7.5	6.6	6.4	7.1	7.5
SQRA	6.0	5.8	5.5	5.1	5.4	5.3	5.2	4.9
QRM	1.7	2.2	3.0	3.4	1.5	1.9	2.3	2.9
SQRM	1.3	1.6	1.9	1.9	1.1	1.2	1.4	1.3
LQRA	2.6	2.7	2.9	3.3	2.1	2.2	2.7	3.1
iQRA	2.0	2.4	2.8	3.4	1.6	1.9	2.7	3.3
iSQRA	1.4	1.6	1.4	1.4	1.1	1.1	1.3	1.3

Variable selection



Practical runtimes

PostForecasts.jl

Per day (24 hours × 99 quantiles) computation time in seconds

QRM	3		
SQRM	3.8		
Ensemble size (M)	**10**	**25**	**50**
QRA	4.6	7.6	12
SQRA	14.8	96.7	473.8
LQRA	118	220.4	456.1
iQRA	3.8	4.2	5.3
iSQRA	8	23	69.2

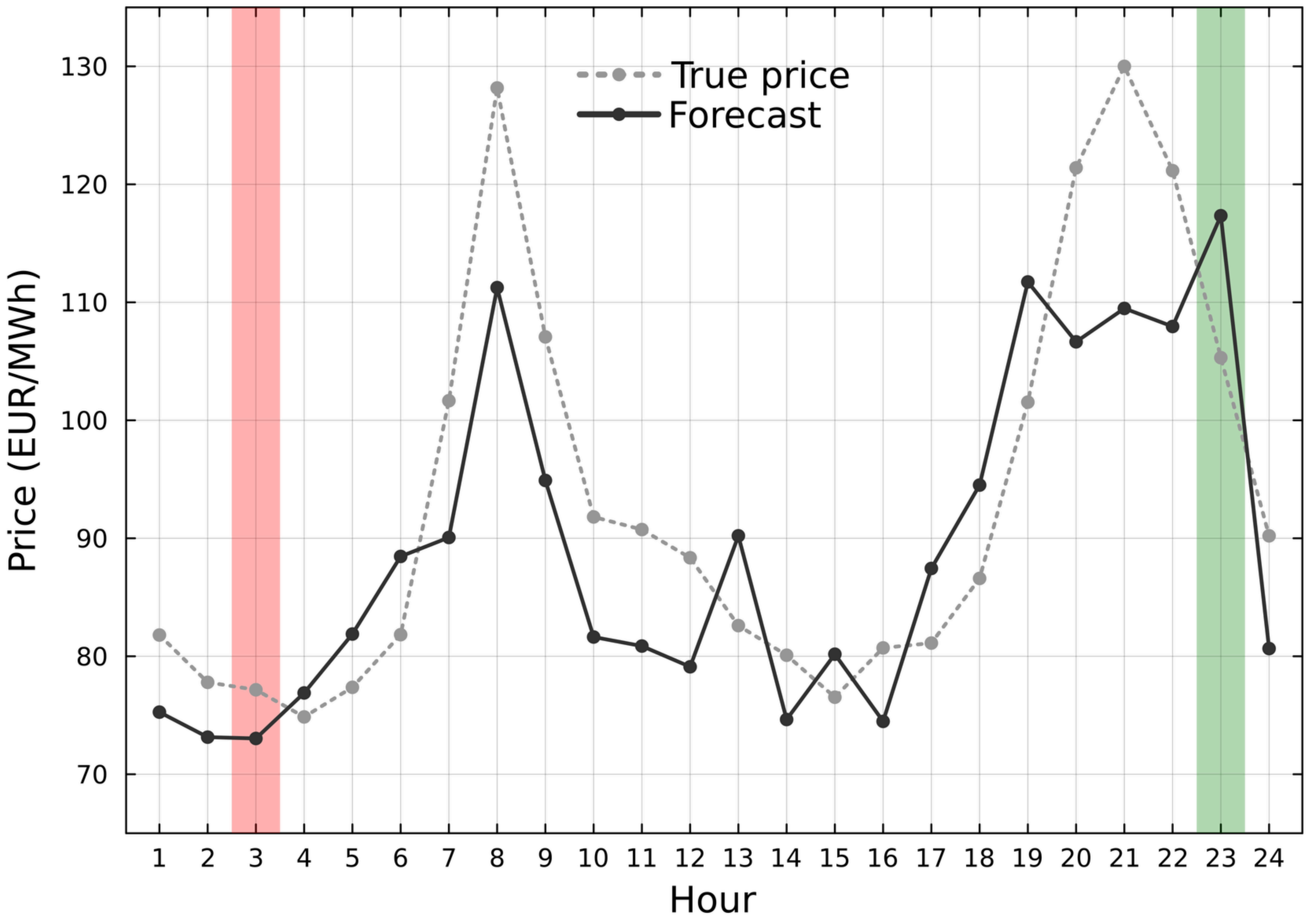
Take home

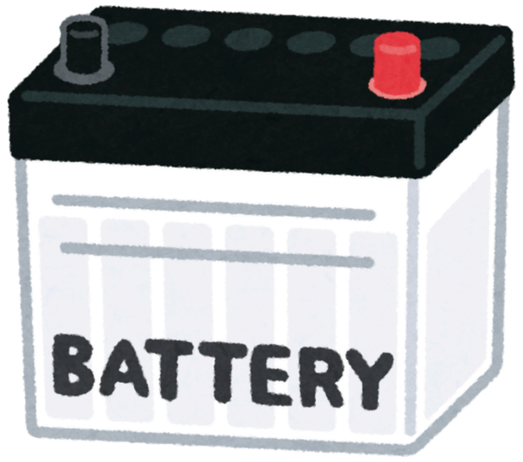
Isotonic constraints in linear quantile regression offer:

- **Higher accuracy**
- **Variable selection without hyper tuning**
- **Lower computational complexity**

- **XGB results**
- **Trading strategy**

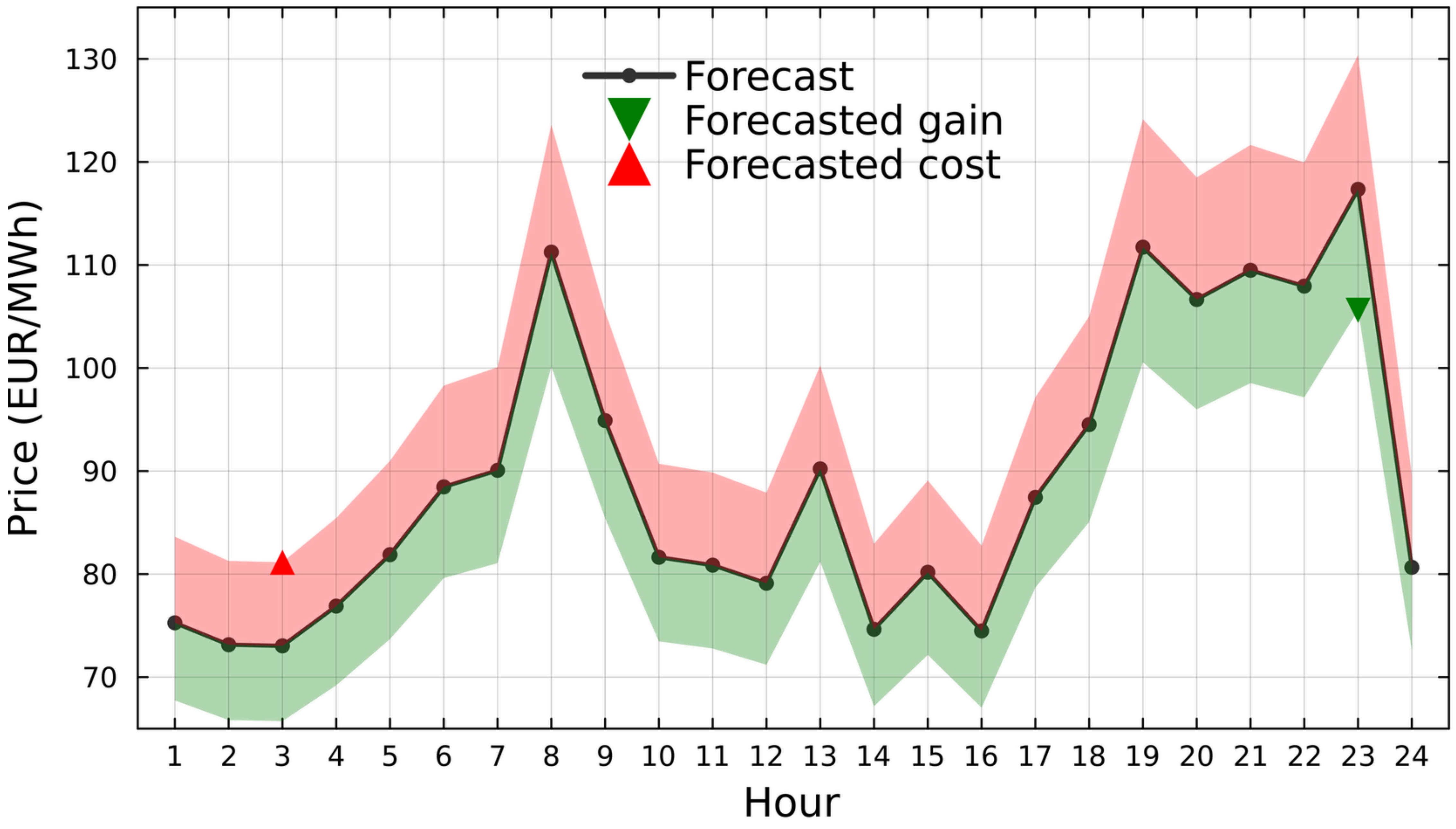
Economic evaluation





Capacity	**1 MWh**
C-rating	**1**
Efficiency	**0.9**

Economic evaluation

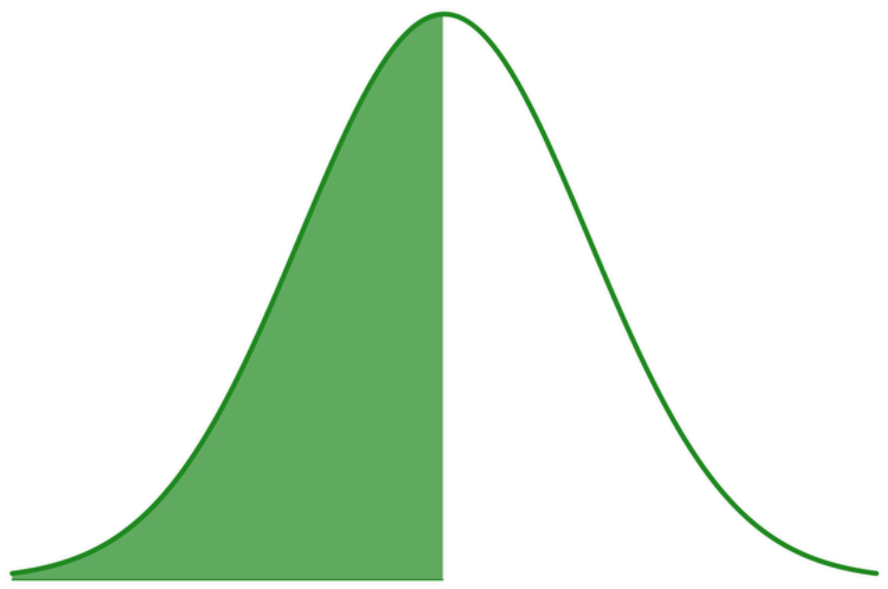


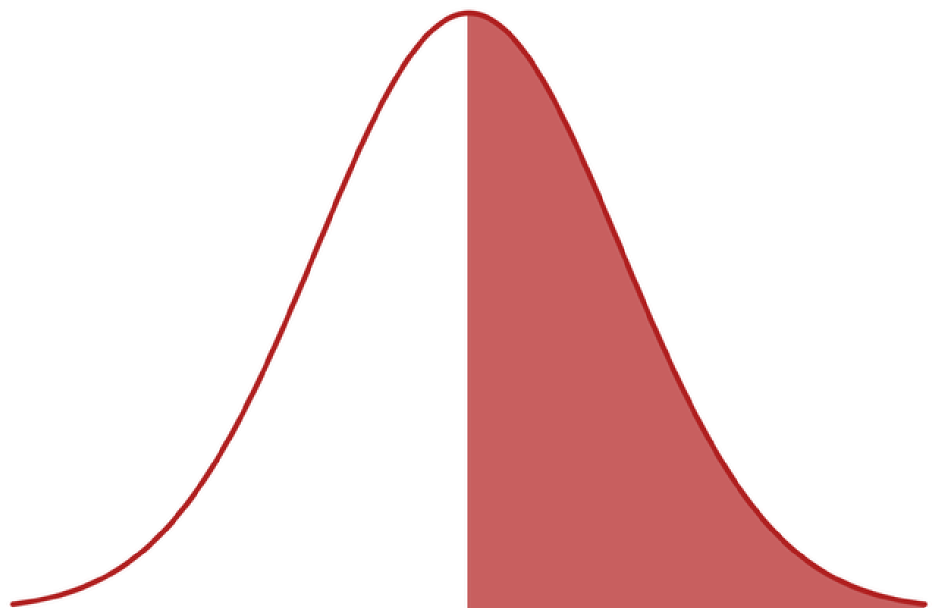
Profit = Efficiency × Sell price − (1/Efficiency) × Buy price

$$\hat{S}_d = \eta \hat{p}_{d,h_s} - \frac{1}{\eta} \hat{p}_{d,h_b}$$

Risk-adjusted trading

$$\hat{S}_d = \eta \hat{p}_{d,h_s} - \frac{1}{\eta} \hat{p}_{d,h_b}$$

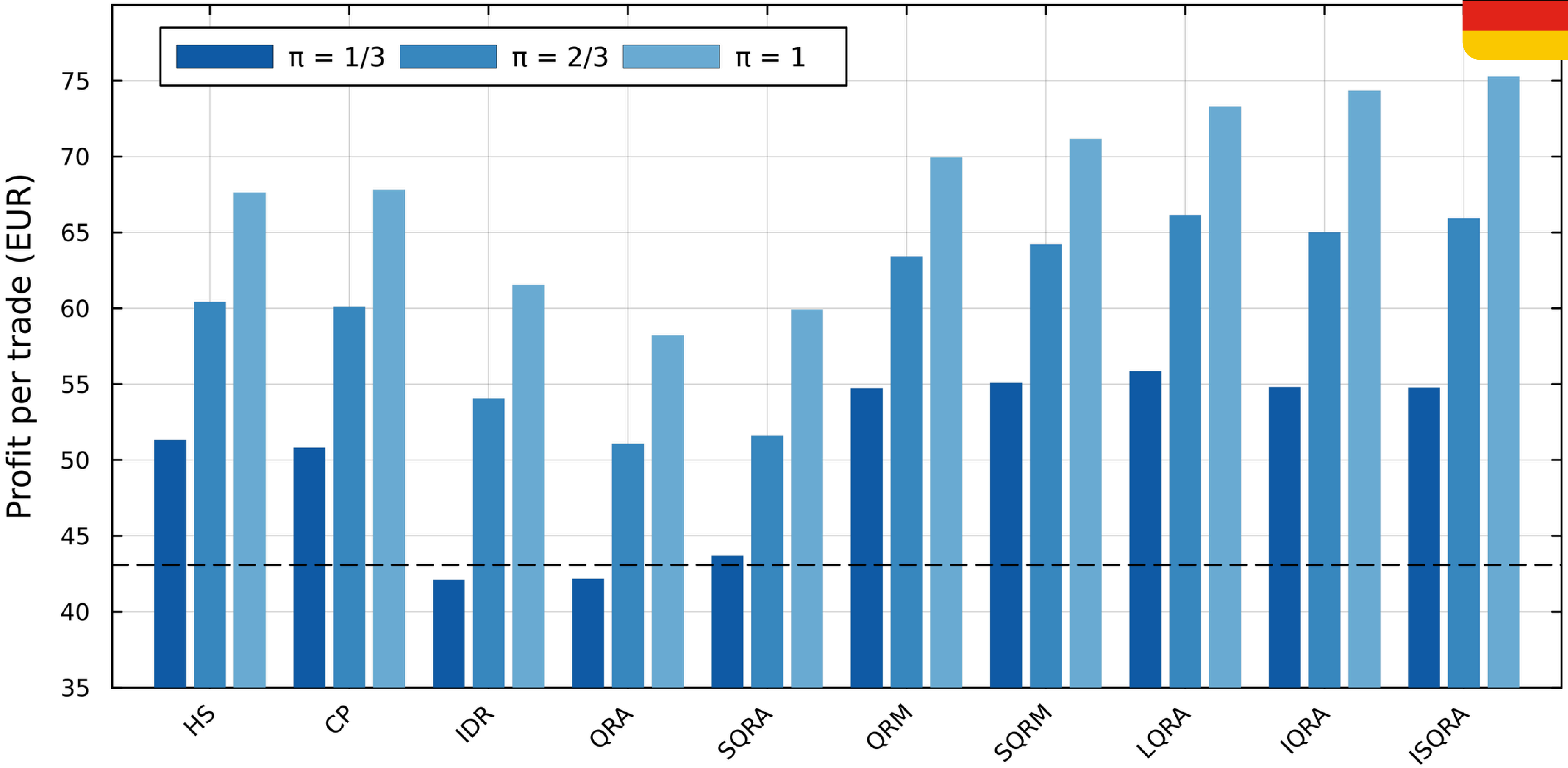




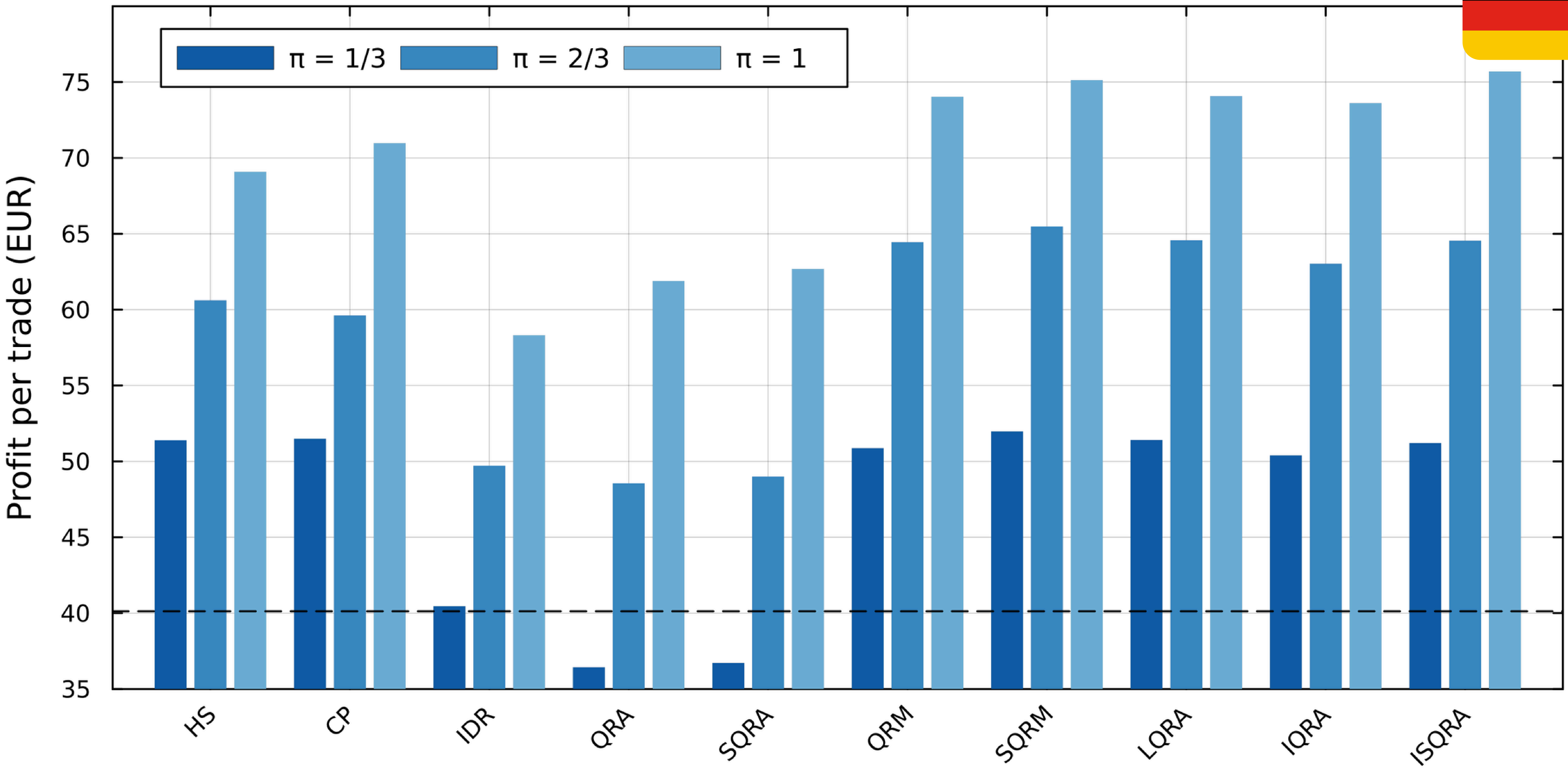
$$R_d = \eta \sum_{i=1}^{49} \left(\hat{P}_{d,h_s}^{0.5} - \hat{P}_{d,h_s}^{i/100} \right) - \frac{1}{\eta} \sum_{i=1}^{49} \left(\hat{P}_{d,h_b}^{1-i/100} - \hat{P}_{d,h_b}^{0.5} \right)$$

Trade if $\hat{S}_d > \pi R_d$

Risk-adjusted trading (NARX)



Risk-adjusted trading (XGB)



CRPS (XGB)

	Germany			France		
M	**10**	**25**	**50**	**10**	**25**	**50**
HS	5.82	5.80	5.79	6.22	6.19	6.19
CP	5.84	5.82	5.81	6.28	6.25	6.25
IDR	5.78	5.78	5.78	6.22	6.22	6.23
QRA	5.64	5.87	6.31	6.10	6.36	6.83
SQRA	5.63	5.85	6.32	6.08	6.34	6.83
QRM	5.62	5.60	5.59	6.04	6.01	6.00
SQRM	5.62	5.60	5.59	6.03	6.00	6.00
LQRA	5.51	5.47	5.46	5.96	5.92	5.91
iQRA	5.53	5.49	5.48	5.97	5.92	5.91
iSQRA	5.53	5.49	5.47	5.96	5.92	5.91

Average pinball for extreme quantiles (XGB)

	Germany				France			
PI	1-99%	2-98%	5-95%	10-90%	1-99%	2-98%	5-95%	10-90%
HS	1.02	1.6	2.84	4.22	1.03	1.64	2.97	4.46
CP	1.02	1.61	2.85	4.24	1.03	1.65	3	4.5
IDR	1.19	1.63	2.69	3.98	1.3	1.8	2.93	4.31
QRA	1	1.47	2.59	3.97	1.13	1.64	2.87	4.35
SQRA	1.02	1.48	2.58	3.95	1.14	1.65	2.85	4.33
QRM	0.84	1.35	2.49	3.85	0.85	1.39	2.61	4.07
SQRM	0.84	1.34	2.49	3.84	0.84	1.38	2.61	4.07
LQRA	0.74	1.19	2.26	3.61	0.85	1.33	2.48	3.91
iQRA	0.77	1.24	2.32	3.64	0.82	1.33	2.5	3.93
iSQRA	0.77	1.23	2.31	3.63	0.81	1.32	2.5	3.93

Average coverage error for prediction interval (XGB)

	Germany				France			
PI	**1-99%**	**2-98%**	**5-95%**	**10-90%**	**1-99%**	**2-98%**	**5-95%**	**10-90%**
HS	1.89	2.69	4.03	4.63	2.15	3.02	4.03	4.41
CP	1.57	2.3	3.8	4.28	1.69	2.5	3.75	3.99
IDR	8.57	8.63	8.49	7.7	8.41	8.43	8.15	7.05
QRA	6.32	6.19	6.41	6.36	6.04	5.95	6.24	6.32
SQRA	5.32	5.13	4.62	3.78	5.26	5.15	4.53	3.84
QRM	1.52	1.95	2.61	2.73	1.5	1.78	2.19	2.38
SQRM	1.16	1.4	1.71	1.38	1.06	1.19	1.27	0.88
LQRA	1.91	1.92	1.75	1.86	1.83	1.87	2.05	1.77
iQRA	1.49	1.82	1.97	1.95	1.44	1.78	2.13	2.19
iSQRA	1.02	1.09	0.72	0.02	0.98	1.07	0.96	0.32